Epistemic uncertainty management in risk assessment: connections between robustness and sensitivity analysis tools

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Uncertainty quantification (UQ) in computer models has become increasingly important over recent decades, particularly in the context of risk assessment. Numerous techniques exist to propagate aleatory uncertainty, enabling the evaluation of risk-oriented quantities of interest, such as low failure probabilities or high-order quantiles. However, these evaluations often rely on subjective assumptions, including the choice of the input joint probability distribution, which may be based on limited information. Therefore, input densities are tainted with *epistemic uncertainties* which have to be taken into account, especially in risk or safety analyses. The core idea is to find a relevant framework to model such uncertainties and to evaluate the robustness of the estimated key risk measures (typically, failure probabilities, quantiles or any other risk measure) with respect to these assumptions on the input probabilistic modeling.

On the one hand, a first solution is to adopt a "sensitivity analysis" viewpoint. More specifically, *robustness analysis* (see, e.g., [1, Chap. 6]) offers a useful approach by quantifying how perturbations in the assumptions impact the key quantities of interest on which industrial decisions are based. Among several methods, the *Perturbed Law-based sensitivity Indices* (PLI) have been proposed by [2] as a way of measuring the impact of perturbations of input densities (in a parametric case) on a risk measure (e.g., a failure probability, a high-order quantile or a superquantile in [3]). More recently, a novel formulation of these indices has been proposed in [3] by revisiting the intitial formulation through an information-geometric approach, leading to a more sound and rigorous framework for the input-perturbation statistical procedure.

On the other hand, the modeling, quantification and propagation is an old topic in the UQ community. Several mathematical frameworks have been proposed and studied. Among others, two are of specific interest here: the *Optimal UQ* framework [5] and the *Info-gap* framework [6]. In Optimal UQ, epistemic uncertainties are handled through solving an optimization problem leading to maximizing a risk measure (e.g., a quantile) over a class of bounded distributions satisfying moments constraints. As for Info-gap, it relies on maximizing the risk measure over increasingly large input uncertainty domains. By looking at those formulations closer, it appears that methodological links can be drawn from the two frameworks and the PLI-based robustess analysis described above. More specifically, connections and differences can be derived from several keypoints such as the a priori assumptions made, the way perturbations/optimization are solved as well as the final results available at the end of each analysis.

Thus, the goal of this work is to exhibit, discuss and analyze these links both theoretically and numerically, in the context of risk-oriented analyses. From a sensitivity analysis perspective, such a work aims at pointing out the fruitful connections that handling "epistemic" or "second-level" uncertainties impose between this field and the UQ practice in engineering. As a perspective, one can wonder whether designing "optimal" sensitivity measures would be possible in order to assess the robustess of any risk measure with respect to prior assumptions on the input probabilistic model.

References:

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