Sensitivity analysis for Bayesian optimization with uncertainties

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This work tackles the challenge of chance-constrained optimization under uncertainties, which entails significant computational burdens in practical applications (such as the robust design of an electrical machine). Such robust optimization problem can be defined as follows

$$x^* = \arg\min_{x \in \mathcal{K}} \mathbb{E}[f(x,U)] \text{ where } \mathcal{K} = \{x \in \mathcal{X} \subset \mathbb{R}^d \text{ s.t. } \mathbb{P}[g(x,U) \leq 0] \geq \alpha\}$$

with x the vector of design variables and U the vector of uncertain variables.

Since the underlying models of f and g usually are costly computer codes, classical methods are out of the table as they often require numerous evaluations of these codes.

Thus, we use instead Bayesian Optimization and, more specifically, rely on EFISUR [1], an adaptation for constrained Bayesian optimization in presence of uncertainties. First, f and g are modeled using Gaussian process regression in the joint design and uncertain variable space and an acquisition criterion that considers both the average improvement in the objective function and the reliability of the constraints is defined. However, high dimensionality in either the design space or the uncertain parameter space can pose challenges due to the complexity of the optimization steps and Gaussian Processes (GPs) fitting.

Among all the different strategies to deal with the limitations of EFISUR in high dimensions, we propose an adaptation through a dimension reduction of the search space by incorporating Sensitivity Analysis in the sequential approach. Sensitivity analysis approaches allow to understand how each input affect the outputs and to mitigate the effects of the curse of dimensionality by retaining only the influential variables. The first important aspect of this work is the development of new sensitivity indices in order to deal with uncertain variables. Indeed, instead of considering how the inputs affect a scalar-valued output, which is already widely addressed in the literature, we measure the influence of uncertain variables by their impact on a set-valued output characterized as

$$U = (U_1, \ldots, U_p) \longrightarrow \Gamma = \{x \in \mathcal{X}, f(x,U) \leq q \text{ and } g(x,U) \leq 0\}.$$

We derive kernel-based sensitivity indices using an appropriated kernel to compare sets with the necessary properties in [2]. Other approaches were also considered on an industrial test-case for comparison [3].

Using existing goal-oriented indices for deterministic variables and these new indices for uncertain variables, different methodological developments based on various strategies of incorporating sensitivity analysis into EFISUR have been proposed. They will be presented in this work, with applications on comprehensive toy functions and a real-life test case of the robust optimization of an electrical machine.

References:

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