

Enhanced Metamodeling and Sensitivity Analysis for Complex Models Using Tree-PCE. Applications to Hydro-Morphodynamics Modeling.

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Hydro-morphodynamic modelling is affected from different sources of uncertainty, which occur in process-based models, such as inaccuracy in the model inputs, errors in model structure (e.g., poorly described or omitted physical processes), and from limited computing resources. This study is motivated by the analysis and the characterization of some of these uncertainties, elucidating the factors contributing most significantly to the variability of the model output by employing Sobol sensitivity analysis indices [1]. In practice, the computation of Sobol indices, which involves the stochastic estimation of statistical moments and sensitivity indices, is commonly performed using the Monte Carlo method. However, this approach can be computationally expensive, and the runtime can be significantly reduced by employing a surrogate model in place of the high-fidelity solver. One such surrogate modeling technique is the Polynomial Chaos Expansion (PCE) strategy [2], which approximates the model output $Y = f(X)$, where $X \in \mathcal{D}_X \subset \mathbb{R}^d$, by a polynomial \hat{Y} of degree P , constructed from a set of polynomial basis functions $\{\Phi_\alpha\}_\alpha$ defined on \mathcal{D}_X , which are orthonormal with respect to the law of the input vector X . The PCE approximation is defined as:

$$\hat{Y} = \sum_{|\alpha| \leq P} y_\alpha \Phi_\alpha(X)$$

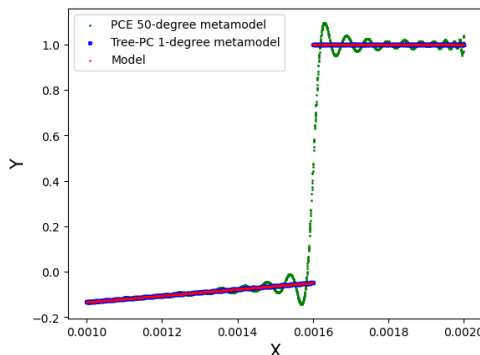
where $\{y_\alpha\}_\alpha$ are the coefficients of Y in the orthonormal basis $\{\Phi_\alpha\}_\alpha$.

The inherent non-linearity of processes in morphological models often causes model outputs to exhibit low sensitivity to input variations until a critical morphological threshold is reached. To accurately represent this variability using a global polynomial approximation, high-degree polynomials would be required, leading to increased numerical complexity. Drawing inspiration from regression trees in supervised learning, we propose an adaptation of the Polynomial Chaos Expansion (PCE) method, called Tree-PCE [3], to address this challenge in complex models. This approach decomposes the input domain into hyperrectangular subdomains, indexed by a binary tree, where local PCE is applied within each subdomain R . The Tree-PCE global metamodel obtained from local metamodels defined by

$$\hat{Y} = \sum_R \sum_{|\alpha| \leq P} y_\alpha^R \Phi_\alpha^R(X)$$

with $\{\Phi_\alpha^R\}_\alpha$ being a polynomial basis orthonormal with respect to the law of X on the subdomain R and $\{y_\alpha\}_\alpha^R$ are the coefficients of Y in the orthonormal basis $\{\Phi_\alpha\}_\alpha^R$.

By minimizing the influence of irregularities within these subdomains, the method enables the use of local low-degree polynomial approximations. The resulting local metamodels effectively capture the model's behavior in each region, significantly improving the representation of complex dynamics. In contrast, a global polynomial model would require a much higher degree to achieve comparable performance. Moreover, a by-product of this approach is an analytical formula allowing the computation of global Sobol indices from the coefficients of the local PCE with almost no additional cost.



Comparison of approximating a discontinuous function by standard PCE and Tree-PCE.

The idea of using local Polynomial Chaos Expansions (PCE) has already been explored in the literature. For instance, works such as El Garnoussi et al. (2020) [4] and Dréau et al. (2023) [5] have proposed techniques to decompose the input domain into subdomains and apply local PCE within each. Additionally, Poëtte and Lucor (2012) [6] introduced an iterative PCE method to enhance the accuracy of non-linear models. However, these approaches have not yet achieved the capability to directly compute Sobol indices from the obtained metamodel coefficients, which is a distinctive advantage of our method.

In conclusion, the proposed method provides a cost-effective solution for uncertainty quantification of complex model behaviors, particularly in hydro-morphodynamic modeling, by enhancing the precision of metamodeling and Sobol indices estimation. The results demonstrate its potential to significantly improve the accuracy and reliability of model predictions, especially when addressing complex dynamics.

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