

Novel Sensitivity Analysis Using SHAPLEY for PROMETHEUS Resilience Modelling

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Abstract:

In the context of increasing global uncertainties, understanding the sensitivity of complex systems is paramount for effective resilience modelling. This study addresses the sensitivity analysis based on the Shapley value, a concept derived from cooperative game theory (Shapley, 1953), which provides a robust framework for evaluating the contribution of individual variables to overall model outcomes.

This study presents the evolution of a resilience modelling approach within the PROMETHEUS project, from its inception using classical statistical methods to the current application of advanced sensitivity analysis techniques. Initially, our model leveraged t-statistics to determine the significance of indicators in predicting resilience outcomes, drawing from a comprehensive set of metrics compiled by institutions such as the European Commission's Joint Research Centre (JRC) and other international bodies (Benczur et al., 2023 and European Commission, 2020).

The PROMETHEUS model's development progressed from logistic regression to linear regression, allowing for a more nuanced understanding of the relationships between various resilience indicators and system outcomes resilience. This transition enabled a more precise quantification of each indicator's impact on overall resilience measures. The following considerations apply:

1. **Logistic Regression (LogRes):** A simple, well-established "expert decision modelling" approach, represented by the formula: $P(Y=1|X) = 1 / (1 + e^{-(\beta_0 + \beta_1 X_1 + \dots + \beta_n X_n)})$ where $P(Y=1|X)$ is the probability of the outcome given the input variables, and β_i are the regression coefficients.
LogRes has limited modulation of the outcomes and assumes independence among independent variables.
2. **Linear Regression (LinRes):** Allowing for a more nuanced understanding of relationships between various resilience indicators and system outcomes, quantified by: $Y = \beta_0 + \beta_1 X_1 + \dots + \beta_n X_n + \varepsilon$ where Y is the outcome variable, X_i are the predictor variables, β_i are the coefficients, and ε is the error term.
LinRes assumes independence among independent variables while our testing has shown strong multicollinearity.
3. **SHAPLEY Method:** An advanced approach that accounts for interactions between variables and provides a fair distribution of contributions among predictors. The Shapley value for variable i is calculated as: $\varphi_i(v) = \sum_{S \subseteq N \setminus \{i\}} (|S|!(n-|S|-1)! / n!) [v(S \cup \{i\}) - v(S)]$ where N is the set of all variables, S is a subset of variables, v is the characteristic function, and n is the total number of variables.
Shapley's advantage is to account for interactions and fair distribution across different independent variables while being computationally intensive ((Iooss, B., & Prieur, C. 2019 and Linkov, I., Trump, B. D., & Keisler, J. 2018).

With this abstract, we aim to introduce the application of the Shapley value method, a concept derived from cooperative game theory, to further enhance our sensitivity analysis. This novel approach allows for a fair distribution of contributions among predictors, enabling researchers to identify key factors influencing resilience in socio-economic systems with predicted higher accuracy.

The Shapley method addresses limitations of traditional sensitivity analysis techniques by accounting for complex interactions between variables. This advancement not only quantifies individual contributions but also captures the intricate interdependencies among variables, leading to more nuanced insights into system behaviour.

The Shapley value allows for a fair distribution of contributions among predictors, enabling researchers to identify key factors influencing resilience in socio-economic systems. By applying this method, we analyse various resilience indicators compiled by institutions such as the European Commission's Joint Research Centre (JRC) and other international bodies. These indicators serve as critical inputs for our model, which aims to predict system responses under different scenarios of external shocks.

The Shapley approach not only quantifies individual contributions but also accounts for interactions between variables and provides complementary insights into model (system) behaviour as compared with traditional variance-based sensitivity analysis (a.k.a Sobol' indices).

Through case studies involving resilience metrics from sectors such as health, environment, and security, we demonstrate how the Shapley method enhances predictive accuracy and informs policy-making.

This research contributes to ongoing efforts to bolster societal resilience against multifaceted challenges by providing a comprehensive analytical tool that integrates empirical data with theoretical foundations (European Commission, 2020). In conclusion, leveraging the Shapley value in sensitivity analysis represents a significant advancement in modelling resilience. This approach not only enriches our understanding of variable interactions but also supports decision-makers in developing strategies that enhance system robustness in an increasingly volatile world.

References:

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