Global Sensitivity Analysis with Optimal Transport: Wasserstein Shapley and the Wasserstein Gap

EMANUELE BORGONOVO
Bocconi University, Department of Decision Sciences, Milan, Italy

ELMAR PLISHKE
Helmholtz-Zentrum Dresden-Rossendorf, Dresden, Germany

In this work, we first review the theory of global sensitivity analysis with optimal transport [1,2,5]. We also review recent applications [3,4]. We show that the associated global sensitivity measures possess several relevant properties, such as zero-independence and max-functionality. The former implies that the global sensitivity measure is zero if and only if the quantity of interest and the input(feature/parameter) of concern are statistically independent. The latter implies that the global sensitivity measure is maximal if and only if the quantity of interest is a deterministic function of the feature of concern. We also show that if the squared Euclidean distance is used in the cost function of the optimal transport, one obtains a decomposition which brings together moment-independent and variance-based indices. In fact, it holds that the distance between the distributions can be decomposed in three terms. The first term equals the individual variance-based contribution. The second term equals the contribution to the output second order moment and the third term accounts for contributions to any higher order moment. We call this third term the Wasserstein Gap.

We then discuss the connection between optimal transport sensitivity and design of experiments, introducing the notion of Wasserstein-Shapley value and discussing the properties of this notion.

References:

[1]E. Borgonovo, A. Figalli, E. Plischke, and G. Savaré. Global sensitivity analysis via optimal transport. *Management Science*, 2024. Accepted.

[2]E. Borgonovo, G. Savaré, A. Figalli, P. Ghosal, and E. Plischke. Convexity and measures of statistical association. 2024. In preparation.

[3]G. Cataldo, E. Borgonovo, A. Siddens, K. Carpenter, M. Nado, and E. Plischke. Global sensitivity analyses for test planning with black-box models in the mars sample return program. 2024. Under review.

[4]L. Chiani, E. Borgonovo, E. Plischke, and M. Tavoni. Global sensitivity analysis of integrated assessment models with multivariate outputs. 2024. In preparation.

[5]J. C. W. Wiesel. Measuring association with Wasserstein distances. *Bernoulli*, 28(4):2816–2832, 2022.

[Emanuele Borgonovo; Bocconi University, via Roentgen 1, 20136, Milan, Italy]
[`emanuele.borgonovo@unibocconi.it` – `www.emanueleborgonovo.com`]