Global Sensitivity Analysis based on Multi-resolution Polynomial Chaos Expansion: Method and Example Application to Coupled Flow Problems

Ilja Kröker
Sergey Oladyshkin
Iryna Rybak
Wolfgang Nowak
University of Stuttgart, Stuttgart, Germany

For decades, global sensitivity analysis (GSA) has been the method of choice for identifying the most relevant or sensitive model parameters in nonlinear modeling [3]. This work focuses on the Sobol indices [9], which belong to the class of the variance-related sensitivity indices. However, despite their long successful history, the classical approach via the Sobol indices uses Monte-Carlo (MC) sampling, typically requiring a high number of model evaluations. Such high computational costs reduce the applicability of the classical MC-based Sobol approaches in applied scenarios, where even individual model evaluations may require substantial computational power.

Surrogate-based techniques, such as polynomial chaos expansions (PCE), can overcome this restriction. Notably, the Sobol coefficients can be computed directly from expansion coefficients, as proposed by Sudret in [10]. However, even PCE-based surrogates have some restrictions. In particular, the classical PCE tends to suffer from Gibbs' phenomenon, which leads to oscillations in the surrogate caused by discontinuities in the model response.

The arbitrary multi-resolution polynomial chaos (aMR-PC) combines two ideas: the data-driven Ansatz of the arbitrary PCE proposed in [8] and the multi-resolution/multi-element based localization initially introduced in the context of uncertainty quantification by Le Maître et al. in [6]. This localization inherently reduces Gibbs phenomena and can achieve higher accuracy without increasing the maximal polynomial degree.

In this work, we extend the concept of the surrogate-based GSA to aMR-PC-based surrogates as proposed in [5]. For demonstration, we apply the extended techniques to a problem taken from the context of porous media. Specifically, we consider fluid flow through a coupled system consisting of a free-flow region and a porous-medium domain. Here, the Stokes equation describes fluid flow in the free-flow domain, and Darcy's law holds in the porous-medium region e.g. [2, 7]. The coupling conditions, ensure the conservation of mass across the interface, the balance of normal forces and use the Beavers–Joseph condition [1] for tangential velocity. The latter contain the parameter characterising pore-space morphology near the fluid-porous interface. Developing and extending such complex models, particularly in the context of model calibration, requires powerful strategies for assessing the relevance of model parameters, for which GSA is an established tool.

In this talk, we demonstrate the application of the aMR-PC-based GSA for the Stokes–Darcy problem, analyzing the sensitivity of four uncertain parameters: the exact location of the interface, the permeability, the Beavers–Joseph slip coefficient, and an geometric uncertainty in the outflow boundary. Furthermore, we use this modeling example to compare aMR-PC and classical PCE-based GSA.

References:

[1] G.S. Beavers and D.D. Joseph, "Boundary conditions at a naturally permeable wall", *J. Fluid Mech.* **30**:197-–207, 1967.

[2] M. Discacciati, E. Miglio, A. Quarteroni, "Mathematical and numerical models for coupling surface and groundwater flows", *Appl. Num. Math.*, **43**:57–74, 2002.

[3] T. Homma and A. Saltelli, "Importance measures in global sensitivity analysis of nonlinear models", *Reliab. Eng. Syst. Saf.* **52**(1):1–17, 1996.

[4] I. Kröker and S. Oladyshkin, "Arbitrary multi-resolution multi-wavelet-based polynomial chaos

expansion for data-driven uncertainty quantification", *Reliab. Eng. Syst. Saf.*, **222**:108376, 2022.

[5] I. Kröker, S. Oladyshkin, I. Rybak, "Global sensitivity analysis using multi-resolution polynomial chaos expansion for coupled Stokes–Darcy flow problems", *Comput. Geosci.* **27**:805–827, 2023.

[6] O.P. Le Maître, H.N. Najm, R.G. Ghanem, O.M. Knio, "Multi-resolution analysis of Wiener-type uncertainty propagation schemes", *J. Comput. Phys.*, **197**(2):502—531, 2004.

[7] K. Mostaf, K. Baber, B. Flemisch, R. Helmig, A. Leijnse, I. Rybak, B. Wohlmuth, "A coupling concept for two-phase compositional porous-medium and single-phase compositional free flow", *Water Resour. Res.*, **47**:W10522, 2011.

[8] S. Oladyshkin and W. Nowak, "Data-driven uncertainty quantification using the arbitrary polynomial chaos expansion", *Reliab. Eng. Syst. Saf.*, **106**:179—190, 2012.

[9] I. M. Sobol, "Estimation of the sensitivity of nonlinear mathematical models", *Mat. Model.*, **2**(1):112—118, 1990.

[10] B. Sudret, "Global sensitivity analysis using polynomial chaos expansions", *Reliab. Eng. Syst. Saf.*, **93**(7):964—979, 2008.

[Ilja Kröker; University of Stuttgart]
[`ilja.kroeker@iws.uni-stuttgart.de` – `https://www.iws.uni-stuttgart.de/institut/team/Kroeker-00002/`]