On one dimensional weighted Poincaré inequalities for Global Sensitivity Analysis

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The principle of global sensitivity analysis (GSA) is to quantify the influence of input variables (viewed as independent random variables) on the output of a multivariate function, often expensive to evaluate. (Total) Sobol indices, although commonly used for this purpose, are numerically expensive to estimate. Through *Poincaré inequalities*, they can be upper bounded by using DGSM (Derivative Global Sensitivity Measures), which are cheaper to compute (see [2]). This makes DGSM cost-effective alternatives for identifying non-influential variables.

In the preprint [1], we develop the use of *weighted Poincaré inequalities* in dimension 1 for GSA. These are similar to the classical ones but include a non-negative weight introduced in the right-hand side of the inequality. The use of weights is sometimes necessary for certain probability distributions that do not satisfy a classical Poincaré inequality (e.g., the Cauchy distribution) and provides an additional degree of freedom to enhance the precision of the upper bounds.

A first work on the use of weights in GSA was proposed in [4]. Their weight is specifically adapted for linear phenomena. Indeed, the underlying weighted Poincaré inequality is saturated (i.e. becomes an equality) for linear functions. We extend their approach by constructing a weight from any suitable monotonic (non-linear) function and developing a numerical method for estimating it. In particular, our algorithm can be used to generate:

- data-driven weights from estimators of the *main effects* (functions representing the individual influence of each variable), when they are monotonic. We establish results on stability and consistency for such weights.
- non-vanishing weights that, somewhat similar to that emphasized in [3], ensure the existence of the so-called *Poincaré chaos* and provide as well sharp lower bounds for total Sobol indices.

We illustrate the relevance of our approach through analytic toy models and a standard application for a simplified flood model (see Figure 1). For instance, Figure 2 displays total Sobol indices, along with their upper and lower bounds, of the four most influential variables – Q (a truncated Gumbell variable), K_s (a truncated normal one), Z_v and H_d (triangular ones)– of the maximal overflow of a river, whose expression is omitted here. For these variables, we compare our results with the unweighted ones derived in [2,3], observing in the weighted cases an important improvement for the upper bounds, as well as a notable gain for the lower bounds.

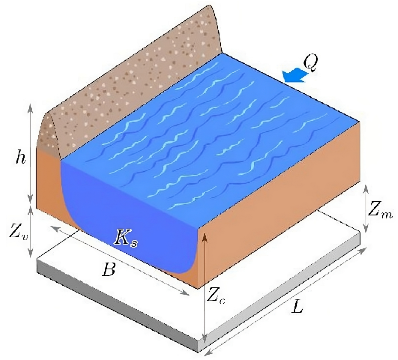

Figure 1: A dyke, a river and variables for flood modeling.

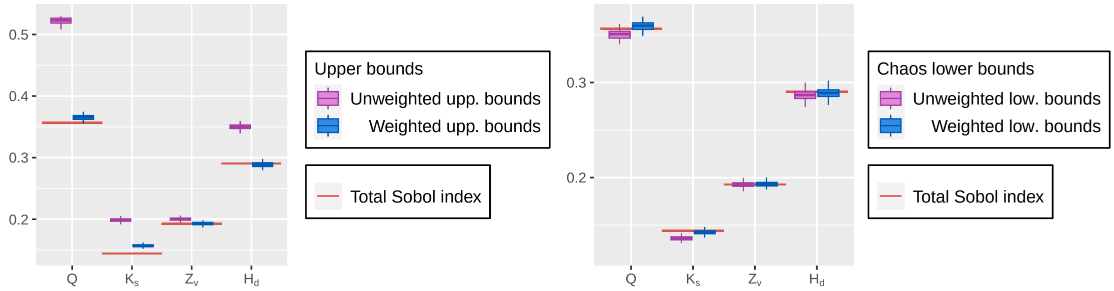

Figure 2: Total Sobol indices and several estimations of their:
(**left**) upper bounds with/without a weight, (**right**) lower bounds with/without a weight, associated with variables Q, K_s, Z_v and H_d in the maximal overflow of a river.

References:

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[3] O. Roustant, F. Gamboa and B. Iooss. "Parseval inequalities and lower bounds for variance-based sensitivity indices", *Electron. J. Stat.*, **14**:386–412, 2020.

[4] S. Song, Z. Tong, L. Wang, S. Kucherenko, and Z. Lu. "Derivative-based new upper bound of Sobol' sensitivity measure", *Reliab. Eng. Syst. Saf.*, **187**:142–148, 2018.

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