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Third Moment Method for Sensitivity Estimation of Failure Probability with respect to Distribution Parameters

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Structural reliability analysis is a crucial part of designing systems that can withstand uncertainty in loading and material properties. Reliability methods provide solutions for evaluating failure probabilities, which can be highly sensitive to input variables. Understanding how changes in input variables affect failure probability is essential for optimization and safety enhancement.

Two kinds of sensitivity analysis of failure probability have been investigated with respect to deterministic and random parameters. The sensitivity measure considered herein is the sensitivity analysis of failure probability with respect to distribution parameters, with the sensitivity index defined as the partial derivative of failure probability with respect to the distribution parameters. Most of existing methods for computing such partial derivative have been developed as the post-processing step of an existing strategy for reliability analysis. Based on first-order reliability method (FORM) and second-order reliability method (SORM), the sensitivity of failure probability is computed with the aid of the so-called design point [1, 2]. Identifying the design point in highly nonlinear problems is challenging, making this sensitivity analysis method unsuitable for such cases. Another approach is simulation methods, such as the crude Monte Carlo Simulation [3], Importance Sampling [4], Lines Sampling [5] and Subset Simulation [6]. A key advantage of simulation methods is that the samples generated for estimating failure probability can be post-processed for sensitivity analysis without the need for additional structural analyses. However, the accuracy of failure probability relies greatly on the quality of sample generation, leading to a significant drop in computational efficiency for small failure probability problems. To decrease the number of structural analyses, surrogate models are adopted with combination of the simulation methods for sensitivity analysis, such as the Kriging model [7]. Although computational efficiency can be enhanced with the help of surrogate models, this introduces new challenges related to the construction of the surrogate model. In summary, the effectiveness of sensitivity analysis methods is significantly affected by the underlying reliability analysis techniques used. The method of moments [8] has been widely used for reliability analysis, demonstrating both efficiency and accuracy in addressing nonlinear problems and those with small failure probabilities. A sensitivity estimation framework based on the method of moments was proposed [9], focusing on methods that utilize the first, second, and fourth moments. Building on this approach, an analytical sensitivity estimation method using the fourth moment has been developed, with inputs modeled as normal random variables.

The present work introduces a third-moment method for estimating the partial derivatives of failure probability with respect to the mean, standard deviation, and skewness of input random variables. Assuming the variables are independent, the sensitivity index is formulated using the third-moment reliability index. An efficient numerical algorithm is developed, enabling the sensitivity index to be computed as a byproduct of the reliability analysis. Numerical examples demonstrate that the proposed method accurately estimates the sensitivity of failure probability with respect to the mean and standard deviation. Additionally, for random variables with small positive or negative skewness, the sensitivity of failure probability with respect to skewness can be reliably estimated.

This study offers three key innovations: (1) It is the first to provide a comprehensive investigation of the third moment method for sensitivity estimation. (2) It includes a detailed numerical algorithm based on the dimension reduction method, where all required inputs are obtained as byproducts of the reliability analysis. (3) It explores the derivative of failure probability with respect to skewness through practical examples.

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