Robust Bayesian Analysis with information geometry and Perturbed-Law based sensitivity Indices

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Abstract

In modern science and engineering, computational models are popular tools for assessing the behavior of physical systems. They may be seen as maps associating some input parameters to quantities of interest (QoI) related to the response of the system under study. Despite their increasing fidelity, computational models remain simplified representations of the real system, and their input parameters are often not perfectly known. As a result, the QoI predicted by such models are tainted with uncertainties.

Bayesian inference constitutes a coherent framework for quantifying uncertainties and updating them by taking into account all available information. This framework is based on a probabilistic description of uncertainties and relies on the definition of a *prior* distribution, which encodes a state of knowledge about some input parameters before making any observations. Then, this prior state of knowledge can be updated through the derivation of a *posterior* distribution, which summarizes all the available information once new data have been observed. In particular, the general framework of Bayesian inference can be applied to inverse problems, in order to update uncertainties of input parameters of computational models from noisy and limited observation data.

The selection of the prior distribution is of utmost importance in the framework of Bayesian inference. This even constitutes a common criticism directed at Bayesian inference. The prior enables the integration of both qualitative and quantitative information related to parameters, including diverse sources such as past experiments, data taken from existing literature, or beliefs of one or several analysts (*i.e.*, expert judgment). Hence, encoding such various information into a single probability distribution appears as a non-trivial task. In this context, the field of Robust Bayesian Analysis, introduced in the early 90s, provides theoretical and computational foundations for the analysis of the influence of the choice of the prior on Bayesian inference results [1,2]. It aims at quantifying the range of variation of a given QoI by assuming that the prior belongs to a set of probability distributions, which represents all the possible choices for the prior.

More recently, a new Uncertainty Quantification (UQ) branch, named Robustness Analysis, has emerged in the field of sensitivity analysis [3,4]. It aims at measuring the impact of the choice of an input distribution, by studying variations of a QoI with respect to this choice. In particular, an interesting method is given by Perturbed-Law based Indices (PLI), originally introduced in the field of reliability analysis [5,6]. These sensitivity indices are simply defined by the relative variation of the QoI, for a given perturbation of the input distribution. In the recent literature, PLI have been proposed for various types of QoI, including failure probabilities [5,6], quantiles [4] or superquantiles [3].

We propose to study the influence of the choice of the prior distribution, through the definition of PLI dedicated to Bayesian inference. The definition of the proposed PLI is based on the recent work of [4], which provides a formal and coherent framework for perturbing input distributions. Such a framework is based on concepts taken from information geometry, notably the Fisher distance on manifolds of probability distributions.

Furthermore, we show that the proposed PLI can be reformulated as the relative variation of some failure probabilities, by using the so-called BUS (*Bayesian Updating with Structural reliability methods*) framework introduced in [7], which establishes an equivalence between Bayesian inference and a reliability analysis problem. Such a reformulation of the proposed PLI is particularly appealing from a computational point of view, since it allows the use of estimation techniques tailored for PLI of failure probabilities [5,6]. As a result, the proposed PLI are estimated through a reverse importance sampling mechanism [5].

The proposed approach is applied to various Bayesian inverse problems with varying complexity. The results suggest that the proposed Bayesian PLI enable to identify the parameters for which the choice of the prior has a significant impact on Bayesian inference results. Moreover, results underline that the proposed approach remains feasible in the case of Bayesian inverse problems with nonlinear models and possibly high-dimensional inputs.

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