Greedy packing algorithms with relaxation

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Following [1], we consider various relaxations of the greedy-packing algorithm for the construction of nested designs (or design sequence) on a given compact set X. The standard, non-relaxed, greedy-packing algorithm guarantees 50% packing and covering efficiency for each design in the sequence [2]. However, it places many design points close to the boundary of X, and a first form of relaxation aims at countering this effect and relies on boundary avoidance [3]: bounds on packing and covering efficiencies are still available, and an improvement in covering performance is observed in practice.

Relaxation can also include some randomness, with bounds on packing and covering efficiencies that can be set arbitrarily close to 50%. Compared to the now popular determinant point processes, the construction of a design of given size n is straightforward (but its stochastic properties are much more difficult to analyse).

When X is the hypercube $[0,1]^d$, the construction can take projections onto canonical subspaces into account, with the generation of random Latin hypercube (Lh) designs as a special case.

Greedy minimisation of the energy for an isotropic kernel K is also a form of relaxed greedy packing: here, each of the n design points present at iteration n has an influence on choice of the next point x_{n+1}. The kernel can be singular, which induces a strong repulsive property between points. It can also be positive definite and define a correlation function for a random process on X, with Matérn kernels as special cases. When the correlation length tends to zero fast enough, the sequence of nested designs is then asymptotically 50% packing and covering optimal.

Finally, the practical implementation of the methods above requires the usage of a big but finite candidate set where points are selected sequentially, which is sometimes a significant limitation: for example, generating a random n-point Lh design in $[0,1]^d$ requires a candidate set with n^d points, which is prohibitively large. A method is proposed which does not have this limitation and selects the coordinates one at a time for each new design point in the sequence (without any guarantee on the packing and covering efficiencies, however).

References:

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[3] A. Nogales Gómez, L. Pronzato and M.-J. Rendas, "Incremental space-filling design based on coverings and spacings: improving upon low discrepancy sequences", *J. of Statistical Theory and Practice*, **15**:77, 2021 (hal-02987983).

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